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## ONSET OF FLOW OSCILLATIONS IN FORCED-FLOW SUBCOOLED BOILING

by Frank A. Jeglic and Thomas M. Grace Lewis Research Center Cleveland, Ohio



NATIONAL AERONAUTICS AND SPACE ADMINISTRATION - WASHINGTON, D. C. - MAY 1965



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#### SUMMARY

A study was made of the onset of flow oscillations occurring when a subcooled liquid undergoes a phase change under forced-flow conditions in a single-tube boiler. Experimental data were obtained with electrically heated tubes of variable length over a range of fluid velocities, temperatures, and pressures. The experiments were conducted at system pressures of 3 to 100 psia with degassed, demineralized water flowing vertically upward through the heated test tube. Visual studies were made in which boiling was simulated by injecting steam or air through the porous wall of a tube. A mathematical model predicting the necessary criterion for the onset of oscillations was formulated on the basis of local rates of bubble growth and collapse. Good agreement was obtained between the experimental and theoretical results.

#### INTRODUCTION

The study of instabilities occurring during the change of phase of a flowing liquid has been the object of many investigations in recent years. Experimental data show that various types of instabilities can result when boiling exists in a heated channel. High-frequency pressure fluctuations have been observed in subcooled boilers (ref. 1). These are believed to be caused by the vapor voids collapsing in the subcooled liquids. "Flow excursions," first reported by Ledinegg (ref. 2), can result from improper matching of the pump and boiler hydraulic characteristics. An instability may also be reflected in a surface temperature excursion, which is no more than the well-known "burnout" phenomenon, generally defined as the transition from the nucleate to the film boiling regime.

<sup>\*</sup>The information presented herein was part of a thesis submitted by F.A. Jeglic to the University of Notre Dame in partial fulfillment of the requirements for the degree of Doctor of Philosophy, October 1964.

Another type of instability observed in boiling systems results in oscillatory flow behavior. Often the inception of flow oscillations determines the upper limit of the operating conditions of a given system. Consequently, much effort has been made to predict the onset of flow oscillations under various physical conditions. A review of the literature and state-of-the-art is given in references 3 and 4.

The objective of this investigation was to study the onset of flow oscillations in a single-tube forced-flow subcooled boiler. For all cases considered, the fluid bulk temperature was below the saturation temperature at both the exit and the inlet of the boiler. Demineralized, degassed water was used as the boiling fluid. Test sections consisted of vertical electrically heated Inconel X tubes of various lengths. All experimental tests were conducted in the pressure range of 3.0 to 100 psia with a liquid bulk velocity of 2.2 to 8.3 feet per second for various inlet bulk subcoolings. Some special test sections were constructed which permitted flow observation and photography. For some of the visual studies, porous-wall tubing, which allowed boiling simulation by injecting steam or air through the porous wall, was used. Comparison between the results obtained with electrically heated test sections and the results of simulated boiling studies was made. A criterion predicting the necessary condition for the onset of flow oscillations in subcooled boiling is derived theoretically. The theory is based on the rate of growth and collapse of the vapor bubbles. Theoretical results were found to compare well with experimental data of this investigation.

## **APPARATUS**

### General

A schematic diagram of the closed-loop forced-flow-boiling heat-transfer facility is shown in figure 1 and is described in detail in reference 1. The system is pressurized or evacuated through the expansion tank by dry nitrogen gas or external vacuum pump, respectively. All component parts in contact with the test fluid were made of corrosion resistant materials to prevent fluid contamination and scale deposits. The fluid is circulated by a gear pump, which is powered by a variable speed drive. The power to the test section and the preheater is supplied by 250- and 70-kilowatt interchangeable alternating-current transformers, each regulated by a saturable reactor. The reactors provide fine continuous control of the power input. A counterflow heat exchanger serves as the system heat sink. No restriction at the inlet of the boiler was provided for the present series of runs.

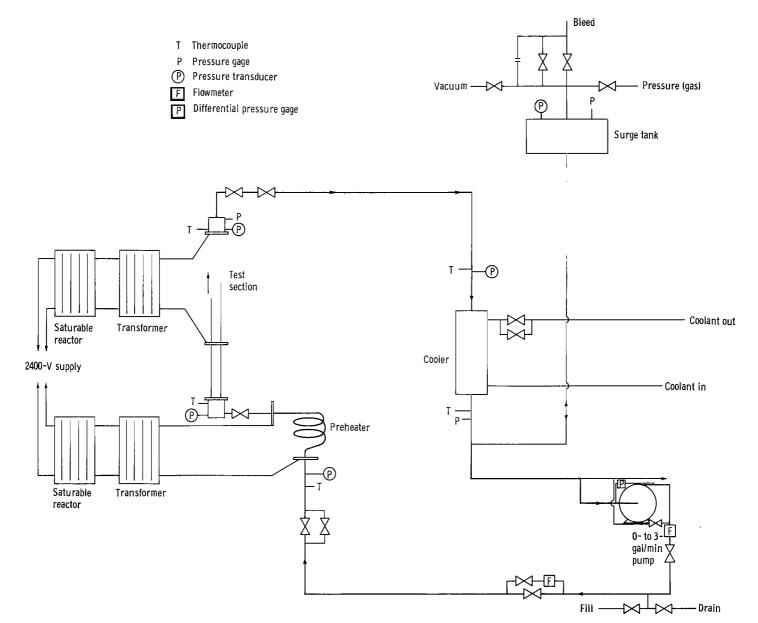


Figure 1. - Schematic of boiling heat-transfer facility.

## **Test Sections**

Electrically heated tubes. - The primary test section was constructed of 0.020-inchwall Inconel X tubing with a 0.25-inch outside diameter. The tubing was especially rolled to limit the wall thickness deviation to less than 3 percent of the nominal wall thickness and had a maximum surface roughness of 20 microinches. The heated length of the test tube was 21 inches (100 diam) and was preceded by a 15-diameter unheated length to reduce the hydrodynamic entrance effects. Power leads were connected to copper flanges that were silver-soldered to the tube. High-temperature solder was used, and care was taken to ensure good contact and to avoid fillets in the heated portion of the test section. The flanges were bolted to the inlet and exit plenums and electrically insulated by Teflon and asbestos gaskets and seals. The exit plenum was fixed to the structural frame, while the inlet plenum was allowed to slide longitudinally to provide for thermal expansion. The test-section assembly was enclosed by a transparent shield to lessen external convective currents and provide protection in case of failure.

A second test section of similar dimensions was built, but it was provided with a sliding copper flange. In this manner the length of the heated portion of the tube could be varied.

To exit plenum direction

Range of camera view

Steam jacket with 0.25-incho. d. porouswall tube

From inlet plenum

Figure 2. - Tubular, porous-wall test section with transparent extension (used in photographic studies).

Transparent test sections. - Three special test sections were built for visual studies. One was constructed of 0.21-inch-inside-diameter porous-wall stainless-steel tubing. A short glass tubing of the same inside diameter was fused to the metallic tube at the exit. The porous-wall tube, with 5-micron-diameter pores, was enclosed by a 1-inch-diameter tube, which formed a chamber connected to a degassed steam or air supply. This test section is shown in figure 2.

Two annular test sections were constructed; the inside tubing was metallic and the outside tubing was glass. In the one shown in figure 3, the inside tube was 0.010-inch-wall, 1/4-inch-outside-diameter Inconel X tubing connected to an electric power supply; in the other, shown in figure 4, the inside tube was 1/4-inch-outside-diameter stainless-steel tubing with a porous wall (5-micron-diam. pores) through which steam could be injected. For both test sections, the inside diameter of the glass tube was 5/8 inch.

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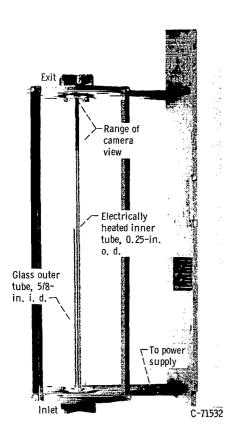


Figure 3. - Electrically heated, transparent annular test section (used in photographic studies).

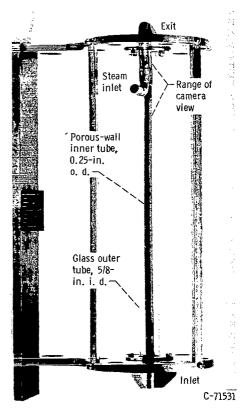


Figure 4. - Annular, transparent test section with porous inside wall (used in photographic studies).

## Instrumentation

The average values of pressures at various locations in the facility were measured with a number of Bourdon tube gages. Fluid bulk temperatures were measured by copper-constantan thermocouples and recorded on a multichannel self-balancing potentiometer. A number of 36-gage Chromel-Alumel thermocouples, spotwelded to the outside surface of the boiler tube, were used to record the surface temperature. A thermocouple spotwelded 1/4 inch from the exit end of the heated length was connected to a separate recording instrument and served as an overtemperature control; that is, an unexpected large temperature excursion would automatically deenergize the power supplies. A dynamometer-type wattmeter was selected to measure the power input to the test section. A multirange, electronic tube voltmeter was used to measure the voltage drop across the tube. The average flow rate was determined by the signal generated by a turbine-

type flowmeter and read on the flow converter and a digital frequency counter. A detailed description of the instrumentation is given in reference 4.

Strain-gage-type transducers with an accuracy of 1/4 percent of full scale were used to measure the instantaneous pressures at various locations of the loop (see fig. 1, p. 3). The signals were continuously recorded on a multichannel oscillograph. The flow transients were obtained by connecting the signal, generated by the turbine flow meter, to the oscillograph. The response time of the flowmeter was 3 to 5 milliseconds, which is sufficient to obtain the instantaneous values of flow over the range of conditions studied herein. One of the thermocouples spotwelded to the wall was also connected to the oscillograph and provided qualitative measurement of the surface temperature variation. Whereas the response of the thermocouple was sufficient to detect the temperature oscillations, the recorded signal was somewhat obscured by the alternating-current pickup from the power supplies.

A high-speed camera, capable of speeds up to 6000 frames per second, was used to photograph the transparent test section during the run in the visual studies. A photocell was mounted next to the test section and its signal recorded on the oscillograph. A light source could thus be recorded simultaneously on the film and the oscillograph. This enabled synchronization of the high-speed movies with the flow and pressure traces.

### **PROCEDURE**

## Preparations

Prior to filling, the system was flushed with deionized water, purged with nitrogen, and evacuated. It was then filled with deionized, deaerated water. The liquid was circulated through the system and further degassed by boiling in the test section and venting to the atmosphere. Water samples were taken and analyzed on a gas analyzer for gas content. Gas content of 3 ppm (by weight) was considered a maximum permissible amount; the degassing process was continued until the air content was well below this value.

The electric resistance of the test-section tubing was measured a priori, and the values obtained agreed well with the electric resistivity given in reference 5. It was also found that for the range of temperatures of this investigation the resistance may be considered independent of temperature.

### Calibrations

All instruments were calibrated prior to installation and were periodically checked. The temperature recording instruments were calibrated with a standard potentiometer. All pressure gages were checked on the deadweight tester. The transducers were mechanically and electrically balanced and checked against a known pressure. Under steady-state conditions, a good correspondence was obtained between the readings of the pressure gages and those recorded on the oscillograph. The flowmeters were calibrated prior to their installation. The flow converter and the digital frequency counter were calibrated by a standard signal generator. The wattmeter and the voltmeter, both used for input power measurement, were calibrated with a known source. Their rms characteristics were checked with known square, triangular, and sinusoidal signals.

The power input calculated from the wattmeter measurements and from the voltmeter reading (together with the known electric resistance of the tube) agreed well in all cases in which both readings were taken. These values were also in good agreement with the calculated enthalpy increase of the fluid, which served as a further check on the power, flow, and bulk temperature readings. When the liquid was preheated and all the heat load removed from the test section, it was possible to check the wall temperature and fluid bulk temperature instrumentation. In effect, therefore, all pertinent instrumentation was continuously checked.

## Experimental Technique

Electrically heated tubes. - The normal operating procedure consisted of setting the flow rate, inlet bulk temperature, and exit pressure to the desired values and then slowly increasing heat flux to the boiler until the oscillations set in or net vapor was generated, whichever occurred sooner. Flow rate and exit pressure were adjusted during the run to maintain the preset value. For the runs with constant amplitude oscillations the system was permitted to oscillate for a substantial period of time before the heat flux was reduced. The runs with increasing amplitude oscillations, however, were terminated by the automatic power shutoff, which was energized by the surface temperature excursion. Any one of the three controlled variables, flow rate, inlet bulk temperature, or exit pressure, was then changed and a new run begun. All recording instruments were turned on before the onset of oscillations and left on for the duration of the run.

Once the heat flux corresponding to the incipience of oscillations was determined for a given set of conditions, the mode of operation was changed for a few runs to determine its effect on experimental results. Either the flow rate or pressure was raised initially, and the heat flux was set at the value previously found to induce oscillations, whereupon

### TABLE I. - EXPERIMENTAL DATA WITH CONSTANT LENGTH,

#### ELECTRICALLY HEATED TUBE

[Tube inside diameter, 0.21 in.; length, 21.0 in.]

Run	Inlet bulk	Exit bulk	Exit pressure,	Inlet liquid	Heat flux (at onset
	temperature,	temperature,	psia	velocity,	of oscillations),
	$^{\mathrm{o}}\mathbf{F}$	<sup>o</sup> F	(a)	ft/sec	Btu/(sec)(sq ft)
<sup>b</sup> 20 - 1-1	79	222	20. 0	2.20	49. 1
<sup>b</sup> 20 - 1-2	148	223		2.20	34. 1
<sup>c</sup> 20 - 1-3	175			2.20	
<sup>b</sup> 20 - 2-1	68	211		4.00	93.5
<sup>b</sup> 20 - 2-1a	67	208		4. 00	91. 7
<sup>b</sup> 20 - 2-2	94	216		4. 00	79. 0
<sup>b</sup> 20 - 2-3	115	219		4. 00	68. 5
<sup>b</sup> 20 - 2-4	146			4.00	55.8
<sup>c</sup> 20 - 2-5	183			4.00	
<sup>d</sup> 20 - 3-1	61	211		6.25	146.0
d <sub>20 - 3-2</sub>	91	220		6.25	129
<sup>d</sup> 20 - 3-3	108	222		6. 25	113
<sup>d</sup> 20 - 3-4	130			6. 25	94.7
e <sub>20 - 3-5</sub>	150			6.25	87.6
<sup>c</sup> 20 - 3-6	182			6.25	
d <sub>20 - 4-1</sub>	64	219		8. 25	201
<sup>d</sup> 20 - 4-2	99	217		8. 25	161
f 20 - 4-3	124		<b>Y</b>	8. 25	
g <sub>40</sub> - 1-1	80		40.0	2.20	75. 6
<sup>c</sup> 40 - 1-2	152		40. 0	2.20	
b <sub>40 - 2-1</sub>	70	266	40. 0	4. 00	125
<sup>c</sup> 40 - 2-2	100		40. 0	4. 00	
<sup>b</sup> V - 1-1	80	140	3.0	2.20	19. 7
<sup>b</sup> v - 2-1	71	140	3.0	4.00	39. 0
<sup>d</sup> V - 3-1	70	140	3.0	6.25	62.5
<sup>d</sup> V - 4-1	70	140	3. 0	8. 25	85. 6

<sup>&</sup>lt;sup>a</sup>For all runs at 60 and 100 psia exit pressure, over the same range of liquid velocities and maximum inlet subcooling, net quality was easily obtained without observing any oscillatory behavior in the subcooled regime.

 $<sup>^{\</sup>mathrm{b}}\mathrm{Constant}\text{-amplitude}$  oscillations in flow, pressures, and surface temperature.

<sup>&</sup>lt;sup>c</sup>Net quality vapor easily obtained.

<sup>&</sup>lt;sup>d</sup>Oscillations with increasing amplitude that resulted in temperature excursion.

<sup>&</sup>lt;sup>e</sup>Irregular oscillations apparent; net quality easily obtained.

f No oscillations detected in subcooled regime; net quality easily obtained.

gIrregular oscillations apparent; net quality easily obtained.

the flow or pressure was reduced slowly until the system was rendered unstable. It was found that the onset of oscillations was independent of the mode of operation for the range of this investigation; consequently, the instability was obtained by increasing the heat flux for all subsequent testing.

A similar procedure was followed for the test section of variable length, except that the inlet temperature and exit pressure were not varied. Instead, the length of the test section was changed between the runs.

<u>Visual studies</u>. - As before, the flow rate, inlet bulk temperature, and exit pressure were set to the desired values. With electrically heated test sections, the heat input was slowly increased until oscillations were recorded on the oscillograph. The high-speed movie camera was then activated as well as the light source. Most of the runs were taken at 1000 frames per second. For some runs the heat flux was reduced during the run in order to observe the stabilization of the system.

The same variables were controlled with test sections in which two-phase flow was obtained by steam injection through the porous tube wall. In these studies, steam or air flow rate was increased slowly instead of electric power. The steam flow rate was either kept constant, decreased, or increased during the run, depending on whether it was desired to observe the neutral oscillations, the development of the oscillations, or the destruction of the oscillations, respectively.

## **EXPERIMENTAL RESULTS**

## **Electrically Heated Tubes**

The experimental results obtained with a constant length, electrically heated tube are given in table I. Each series of runs consisted of a number of runs at constant exit pressure and inlet bulk velocity, but at decreasing inlet subcooling. When the inlet subcooling was such as not to cause the flow to oscillate before net quality was being generated, the series was terminated; the preheater load was then reduced to zero, the exit pressure or fluid mass rate was fixed at a new value, and a new series was begun. The values given in table I correspond to the conditions at the onset of oscillations, when these occurred in the subcooled boiling regime.

As shown in table I, apparently different types of oscillations were recorded in subcooled boiling:

(1) During operation at steady-state conditions, an incremental increase in heat flux induced flow, pressure, and surface temperature oscillations. These developed quickly and reached maximum amplitude in a few cycles. The oscillatory condition could then be maintained indefinitely. Typical traces are shown in figure 5, where the variations of

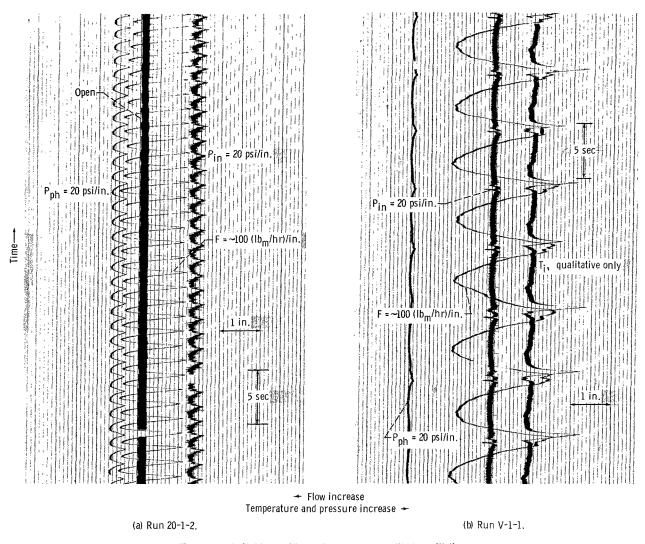


Figure 5. - Typical traces illustrating constant-amplitude oscillations.

pressures and flow are given as functions of time. Further increase in heat flux would result in either surface temperature excursion or net vapor generation.

- (2) During operation at steady-state conditions, an incremental change in heat flux induced oscillations with increasing amplitude and would result in surface temperature excursion. An example of the surface temperature, pressures, and flow variation with time is shown in figure 6. It is believed that these oscillations would reach a constant amplitude if the heat capacitance of the wall could be increased so that the excursion could be decreased sufficiently to prevent the power trip.
- (3) Irregular flow oscillations occurred, but their inception is difficult to define because of their relatively small amplitude. A further increase in heat flux would result in net vapor generation.

The experimental results show that for the range of this investigation the onset of

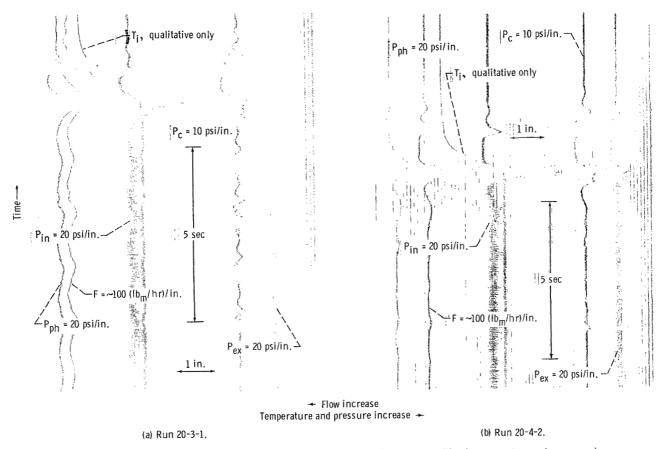


Figure 6. - Typical traces illustrating oscillations with increasing amplitude and resulting in surface temperature excursion.

flow oscillations in forced-flow subcooled boiling is a strong function of inlet subcooling and system pressure. An increase in inlet subcooling or a decrease in system pressure tends to induce subcooled boiling oscillations. It was found that when the inlet bulk temperature and/or exit pressure reached some critical value the oscillations could not be obtained in subcooled boiling regime; under such conditions net quality vapor could be generated at the exit. It is quite possible that in boiling studies under elevated pressures this minimum required pressure was exceeded, and consequently the subcooled boiling oscillations were not observed.

The data obtained with a variable length, electrically heated tube are given in table II. For these runs the exit pressure and inlet temperature were held constant, and the length of the test section was varied. The results indicate that the decrease in the boiler tube length tends to decrease the exit bulk temperature at which the flow oscillations will develop. It should be noted that, whereas the total enthalpy rise of the fluid decreases with decrease in length, the heat flux actually increases.

#### TABLE II. - EXPERIMENTAL DATA WITH VARIABLE

#### LENGTH ELECTRICALLY HEATED TUBE

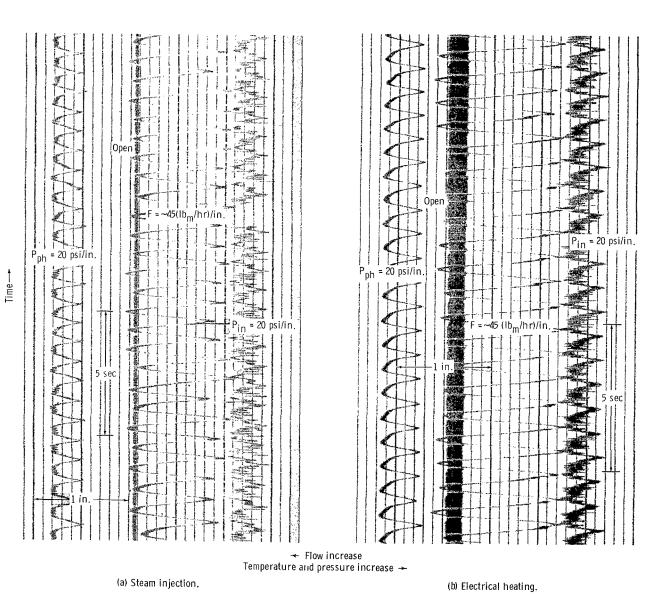
[Tube inside diameter, 0.21 in.; exit pressure, 20.0 psia; inlet liquid velocity, 2.20 ft/sec.]

Run	Test-section length, in.	Inlet bulk temper- ature, <sup>O</sup> F	Exit bulk temper- ature, <sup>o</sup> F	Heat flux (at onset of oscillations), Btu/(sec)(sq ft)
1-1	18. 25	72	216	60.8
1-2	14.0	70	211	76.8
1-3	10. 5	72	214	98.2
1-4	10.0	70	208	104
1-5	7.0	70	199	147
1-6	4.0	72	189	234

### Visual Studies

Initially, for visual studies, the porous-wall tube with a glass extension was used. Liquid flow rate, pressure, and inlet temperature were adjusted to values for which data had been obtained with the electrically heated test section. The rate of steam injection, through the porous walls of the tube, was then slowly increased until flow oscillations were recorded on the oscillograph, whereupon the movie camera was activated. The steam injection rate was not measured, but it was possible to calculate the heat input by the enthalpy increase of the fluid. The correspondence of conditions with the two test sections was excellent; not only did the oscillations occur at the same values, but also the traces were similar. A comparison of the recorded traces is illustrated in figure 7. The flow regime was observed to be bubbly at steady-state conditions, just preceeding the onset of oscillations. As a small increase in steam injection rate resulted in recording oscillations on the oscillograph, a vapor slug appeared in the transparent portion of the test section. The liquid-vapor interface along the wall at the beginning of the slug was quite wavy and irregular. As the slug proceeded up the tube, the liquid film on the wall became increasingly thinner with the interface continually growing smoother. The vapor slug ended abruptly and was followed by bubbly two-phase flow. Immediately behind the vapor slug the bubbles were small and few but slowly grew in size and number until a new slug appeared. The length and the frequency of the slugs depended on the experimental conditions. However, for all cases the frequency of vapor slug formation corresponded with the frequency of the pressure and flow oscillations recorded on the oscillograph. A sample trace is shown in figure 8 (p. 14). The effect of steam-





 $\label{thm:comparison} \textbf{Figure 7. - Comparison of traces obtained with electrically heated test section and with steam injection.}$ 

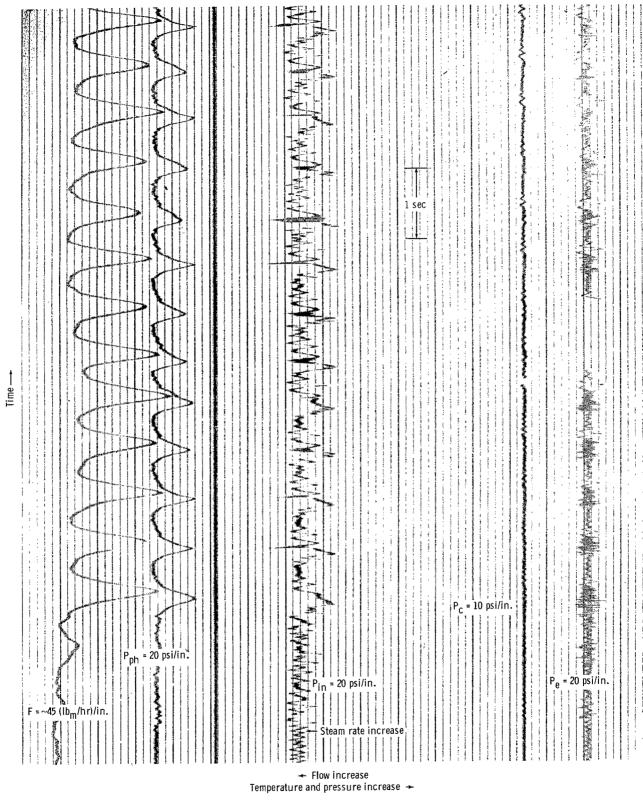


Figure 8. - Typical trace obtained with tubular, porous wall test section.

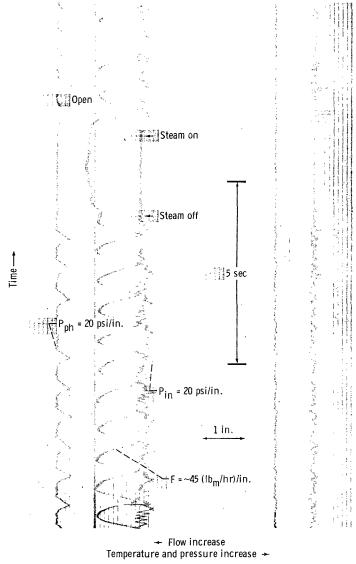


Figure 9. - Effect of steam-injection-rate increase and decrease on stability in simulated subcooled boiling.

injection-rate decrease and subsequent increase is illustrated in figure 9. It can be seen that an increase in steam injection rate can cause a rapid development of oscillations, while decreasing the steam rate results in a stable system.

The high-speed movies were carefully examined and compared to the corresponding traces. From such a comparison it was concluded that, for every cycle, the bubbly flow regime existed when the flow was at the maximum. The flow began to decrease just before the head of the vapor slug appeared in the transparent portion of the tube, continued to decrease as the slug grew and progressed toward the exit plenum, and reached a minimum before the appearance of the tail end of the vapor slug. A complete cycle is shown in figure 10, where the pictures, obtained with the high-speed movie camera, are superimposed on the trace. It should be noted that figure 10 represents one cycle of the run given in figure 8. The photographs represent approximately a 1-inch length of the transparent por-

tion of the tube, which is immediately downstream of the simulated boiling portion of the tube. Even though these movies do not show the actual development of the vapor slugs, they verify that flow oscillations and the ''slugging'' appeared simultaneously.

In a few runs, air instead of steam was injected through the porous wall of the tube. This represented an adiabatic and nearly an isothermal two-phase flow. The most favorable conditions for oscillatory flow were selected, yet over the full range of air injection rate the system remained stable. As the air injection rate was increased, the flow regime changed from bubbly to wavy annular to smooth annular. No slugging flow pattern could be obtained. This was an important observation which led to the conclusion that a

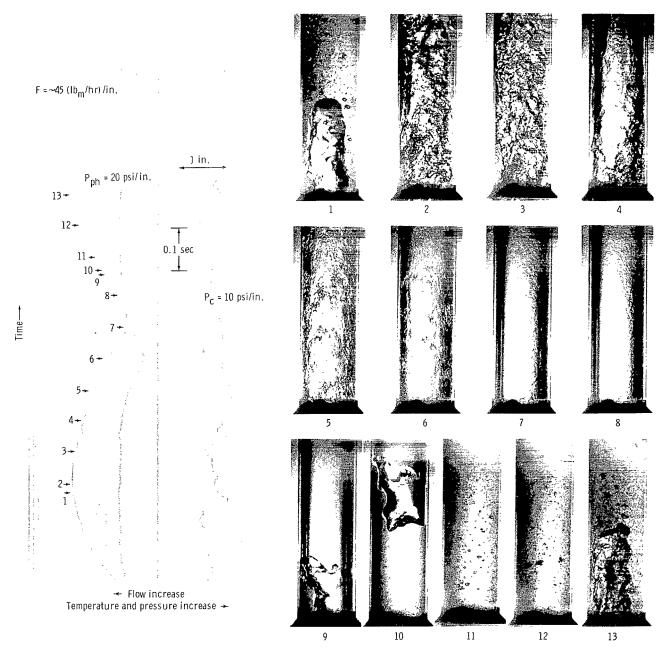


Figure 10. - Two-phase single-component flow behavior under oscillatory conditions obtained with tubular, porous-wall test section.

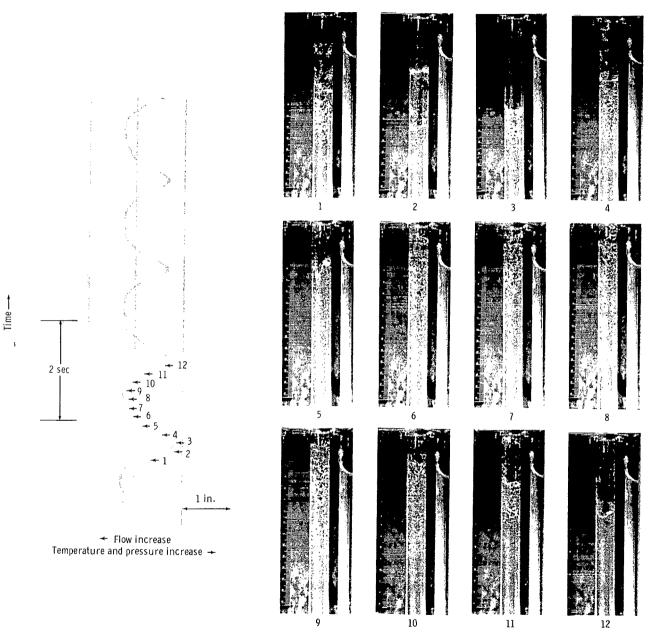


Figure 11. - Two-phase single-component flow behavior under oscillatory conditions obtained with annular, porous-wall test section.

two-component two-phase system will not result in oscillatory behavior in the range of this investigation. Thus, the analogy between single-component and two-component two-phase systems, so often employed, could lead to erroneous conclusions.

The fluid flow pattern behavior at the inception of oscillations in subcooled boiling was visually examined in the two annular test sections, one of which was electrically heated while the other used steam injection through a porous wall to simulate boiling (see figs. 3 and 4 (p. 5)). The preliminary results obtained by the two methods agreed qualitatively as well as quantitatively. It was observed visually that, at the inception of oscillations, there is an abrupt transition from bubbly to annular flow at the exit of the test section. This is illustrated in figure 11. The transition is so sudden that an interface between the two flow patterns can easily be defined, the annular flow portion giving the appearance of a vapor slug. This "interface" oscillates up and down; the frequency of oscillation agrees well with the frequency recorded on the oscillograph.

#### ANALYSIS

When a subcooled liquid is brought in contact with a heating surface, certain temperature gradients will be established within it. If the heating surface temperature is above the saturation temperature, vapor voids may be formed at the surface. These voids may be "torn away" from the surface by one or a combination of the existing forces and condense in the cold liquid.

When a subcooled boiling liquid flowing through a heated channel is considered, the radial temperature profile through the fluid will in part determine how the vapor voids formed at the wall will grow or diminish in size as they penetrate into the lower temperature fluid away from the heated surface. If the conditions are such that the vapor voids emitted from the wall quickly decrease in size and finally collapse, the two-phase flow pattern will give the appearance of bubbly flow. However, if the vapor voids cannot collapse in the cold liquid for one reason or another, it is feasible for these voids (or bubbles) to agglomerate and form larger voids, until they finally give the appearance of a vapor slug at some point in the boiling channel.

As discussed previously, it was observed experimentally that in subcooled boiling the onset of oscillations was simultaneous with the appearance of the slug flow pattern in the boiler tube. When the flow was oscillatory, an oscillating well-defined "interface" between different flow regimes was observed.

Thus, it is postulated that for oscillatory flow behavior it is necessary for the radial temperature distribution in some section of the boiler tube to be such as to permit the bubbles formed at the heated surface to grow. Obviously, the bubbles in the subcooled liquid at the center of the tube will tend to decrease in size, while those in the high-

temperature boundary layer will tend to grow. If the combined effect of the boundary layer and the cold liquid outside the boundary layer results in net growth of the vapor bubbles, then and only then is it possible for the bubbles to agglomerate and form a vapor slug. The formation of the slug would in turn satisfy the necessary condition for the inception of oscillations. Therefore, if the necessary criterion for the inception of oscillations in subcooled boiling is to be found, it becomes imperative to determine the radial temperature distribution and examine its effect on the bubble growth.

At the inception of subcooled boiling, the vapor voids formed at the heated wall are small and few because of the relatively thin thermal boundary layer and the high liquid bulk subcooling. The bubbles that escape from the heated wall quickly collapse in the cold liquid. It would then be expected that in this region the bubbles would have only a small effect on the hydrodynamic behavior of the fluid. Experimental evidence shows that the bubbles remain small for some time after the inception of subcooled boiling, even at very low pressures. As the bulk temperature of the fluid is further increased downstream in the tube, a point is reached where the bubble growth becomes of great importance. This region is developed quickly, as can also be deduced from the theoretical bubble growth analysis, which proves the bubble growth to be an exponential function. Whereas the hydrodynamic and bubble growth phenomena are in reality coupled, it may be postulated that initially the liquid hydrodynamic phenomena are of prime importance, while the bubble growth effects control the fluid behavior in the last stages of the subcooled boiling regime.

Thus, when the fluid behavior is strongly dependent on the bubble growth across the tube, the combined effect of the boundary layer and the cold liquid outside the boundary layer results in some net growth of the vapor bubbles. This condition, however, was postulated to be the necessary condition for vapor slug formation, that is, for inception of oscillations. Then, if the transition from the hydrodynamic to the bubble growth region is relatively short, the thermal behavior of the hydrodynamic region in which the bubble growth has a negligible effect establishes the necessary condition for inception of oscillations. Therefore, in the analysis to follow, the temperature distribution across the tube was determined by neglecting the effect of bubbles; that is, the fluid was assumed to be in the liquid phase. After the temperature distribution was obtained, its effect on bubble growth was examined.

## Temperature Profile

The problem of heat transfer across turbulent incompressible boundary layers has been dealt with by many authors. A discussion of various analyses and their limitations is presented in reference 6. An analysis for obtaining velocity and temperature

distributions in a tube with turbulent flow that applies well for water and takes into account the variation of viscosity with temperature is the investigation of Deissler (refs. 7 to 10). The results of Deissler, which are applicable only for fully developed turbulent flow, were used in the present work to predict the temperature profiles. Deissler's analysis was used primarily for convenience and because of its applicability to water and the present experimental conditions. Its development is briefly reviewed in appendix B.

As shown in appendix B, the temperature profile can be calculated from the equation (B7), which can be rewritten as

$$t = (1 - \beta t^*)t_i \tag{1}$$

where

$$t_i = 41.0 \left(\frac{q_i D_i}{k}\right)_b (N_{Re})_b^{-0.8} (N_{Pr})_b^{-0.4} + t_b$$
 (2)

(All symbols are defined in appendix A.) Equation (2) is the well known Dittus-Boelter equation, which is applicable to fully developed turbulent flow through tubes.

### **Bubble Growth**

Determining the growth of the vapor bubble has been the objective of many investigations in recent years. Scriven (ref. 11), Forster and Zuber (ref. 12), and Plesset and Zwick (ref. 13) have all obtained solutions to the problem, all of which are in good agreement, even though the method of analysis was different. Very recently, Yang and Clark (ref. 14) arrived at the following simple solution for bubble growth constant  $\xi$  by the source theory:

$$\zeta = \frac{\sqrt{\pi}}{2} \frac{\rho_{\ell}^{c} p, \ell}{\rho_{v} h_{fg}} (t - t_{s})$$
 (3)

where  $\rho_{\ell}$ ,  $\rho_{v}$ ,  $c_{p}$ ,  $h_{fg}$ , and  $t_{s}$  are densities of liquid and vapor, specific heat, latent heat of evaporation, and interface temperature, respectively. Equation (3) was derived for the asymptotic stage of the bubble life, which is the time of interest in the present analysis. The results of Yang and Clark are in good agreement with other theories, as discussed in reference 14. As shown in references 11 to 13, the bubble growth in the asymptotic stage may be expressed as

$$R(\theta) = 2\zeta \sqrt{\alpha_{\theta}\theta} \tag{4}$$

where  $\alpha_{\ell}$  is thermal diffusivity and  $\theta$  is time. Differentiation of equation (4) yields the rate of change in the bubble radius:

$$\dot{\mathbf{R}} = \xi \sqrt{\frac{\alpha \ell}{g}} \tag{5}$$

Substituting equation (3) in equation (5) gives

$$\dot{R} = \frac{\sqrt{\pi}}{2} \frac{\rho_{\ell}^{c} p, \ell}{\rho_{v}^{h}_{fg}} \sqrt{\frac{\alpha_{\ell}}{\theta}} (t - t_{s})$$
 (6)

The average bubble growth rate over any cross section of the tube is then

$$\dot{R}_{av} = \frac{\int_0^{r_i} 2\pi r \dot{R} dr}{\pi r_i^2} \tag{7}$$

Substituting equation (6) in equation (7) and assuming the properties to be constant give

$$\dot{R}_{av} = \frac{\sqrt{\pi\alpha_{\ell}}}{r_i^2 \sqrt{\theta}} \frac{\rho_{\ell}^c p, \ell}{\rho_v h_{fg}} \int_0^{r_i} (t - t_s) r dr$$
 (8)

Equation (8) is an expression for the average rate of bubble growth (or collapse) over a cross-sectional area of the tube in which temperature gradients exist. It is evident that if the liquid temperature is greater than the interface temperature  $t_s$  the bubbles will grow; if  $t < t_s$ , the bubbles will tend to collapse.

## **Necessary Stability Criterion**

As discussed previously, the visual experimental studies showed that, in subcooled boiling over the range of this investigation, the flow oscillations resulted simultaneously with the appearance of the ''slugging'' flow pattern in the boiling tube. This observation led to the assumption that the slugging was the necessary condition for onset of flow

oscillations. It was further postulated that slug formation is only feasible when the conditions at some cross section of the tube are such as to yield some net growth of the vapor voids formed at the heated surface. This is equivalent to saying that the right side of equation (8) is greater than 0, or in the limit is equal to 0. Mathematically this may be expressed as

$$\int_0^{r_i} (t - t_s) r dr = 0$$
 (9)

where t is the fluid temperature defined by equation (1) and  $t_s$  is the bubble interface temperature. Equation (9) is then the necessary condition for the onset of flow oscillations for subcooled liquid boiling in a round tube.

## Calculation of Necessary Stability Criterion

The procedure in determining the conditions under which the necessary criterion for stability (eq. (9)) would be met was as follows.

The physical dimensions of the tube, the fluid inlet velocity, and the system pressure were selected such as to correspond to the experimental data. Inlet bulk temperature and heat flux were then arbitrarily chosen. From these values the bulk temperature at the exit was calculated from the heat balance:

$$t_{b, ex} = t_{b, in} + \frac{2q_i L}{c_p \rho r_i u_b}$$
 (10)

For uniform heat flux it is obvious that the most critical case in the tube is at the exit, where the fluid is at the highest bulk temperature. If the necessary stability criterion is to be satisfied at any cross section of the tube, it will first be met at the exit. Therefore, exit conditions were of prime consideration.

Based on the exit bulk temperature, the surface temperature was calculated from equation (2). The initial value of shear stress at the wall was estimated from 1/7 power law, which implemented the calculation of the initial value of the heat-transfer parameter  $\beta$  from

$$\beta = \frac{q_i \sqrt{\frac{\tau_i}{\rho_i}}}{c_p \tau_i t_i}$$
 (11)

For  $y^* \le 26$ , the velocity and temperature parameters were computed as a function of  $y^*$  by simultaneously iterating equations (B3) and (B4). As suggested in reference 10, a value of n = 0.124 was used. For water, for the range of interest, it was found the constant m in equations (B3) and (B4) was m = -1. For  $26 < y^* \le r_1^*$ , equations (B5) and (B6) were used to evaluate the velocity and temperature parameters, when K = 0.36, as recommended in reference 10.

The universal temperature and velocity distributions were thus obtained over the full range of  $y^*$  for the estimated value of  $\tau_i$ . These calculated values were then used in equation (B9), the result of which made it possible to determine a new value of  $\tau_i$  by equation (B8). This new value of shear stress at the wall was used to recalculate the heat-transfer parameter, and the process was repeated until the assumed and calculated shear-stress values were nearly equal. The temperature parameter values were then converted into temperatures by equation (1) and matched with corresponding values of r by

$$r = r_{i} - \frac{\mu_{i}}{\sqrt{r_{i}\rho_{i}}} y^{*}$$
 (12)

The temperature as a function of radius was thus found for a given set of conditions. Before the integral of equation (9) could be evaluated, however, the interface temperature  $t_s$  had to be known. This value was assumed to be the local saturation temperature. Since the pressure was considered constant across the radius in the derivation of equation (9), the saturation temperature was also considered independent of the radius. Hence, with the calculated temperature profile and the assigned interface temperature it was possible to evaluate the integral in equation (9).

A new value of heat flux  $q_i$  was chosen, and the cycle was repeated until the value of the integral changed the sign. The values of the integral thus obtained were plotted against the corresponding values of heat flux, and the result was a value of heat flux at which the integral is equal to 0.

All computations were performed on an IBM 7094 digital computer. Simpson's rule was used for all integrations, with a minimum number of increments being regulated by a maximum allowable error of 0.1 percent. All convergence tests were based on 0.01 percent error. The program was debugged and checked by results obtained independently on an IBM 610 computer. The range of variables used in the calculations was limited to the range of experimental data. The variables used included pressure, velocity, bulk temperature, and boiler length. The results are discussed in the next section.

## 60 50 Unstable 40 0 30 Stable · 20 (a) Bulk velocity, 2, 20 feet per second, 100 Experimental data 0 Heat flux, qi, Btu/(sec)(sq ft) 90 Equation (9) 80 70 Ò 60 0 Unstable 50 40 Stable 30 20 80 100 120 140 160 180 Inlet bulk temperature, <sup>O</sup>F

Figure 12. - Comparison of experimentally determined and theoretically predicted necessary heat flux for onset of oscillations as function of inlet bulk temperature. Exit plenum pressure, 20 psia.

(b) Bulk velocity, 4.00 feet per second.

## DISCUSSION OF RESULTS

## Effect of Inlet Bulk Temperature

In calculating the heat flux that satisfies the necessary criterion for stability (eq. (9)), inlet bulk temperature was taken first as the independent variable. The computed necessary heat flux is plotted against inlet bulk temperatures in figure 12 for four different velocities. The experimental data, given in table I (p. 8), are superimposed on the graphs for comparison purposes. Good agreement between experimental and analytical results is obtained, except at high inlet bulk temperatures. This means that over the full length of the tube the bulk temperature of the fluid in this case is relatively closer to the saturation temperature. Consequently, the conditions are more favorable for formation of larger and more numerous vapor voids at the heated surface. Thus, in this region the bubbles become of increasing importance, which violates the single-phase assumption postulated in the analysis.

Furthermore, when the inlet subcooling is small, the heat input necessary for net vapor generation at the exit is much lower than that for large subcooling. Therefore, the axial rate of change of void fraction would be expected to be less, resulting in a transition from bubbly to annular flow pattern, as depicted in figure 13(a). It is speculated, however, that if the

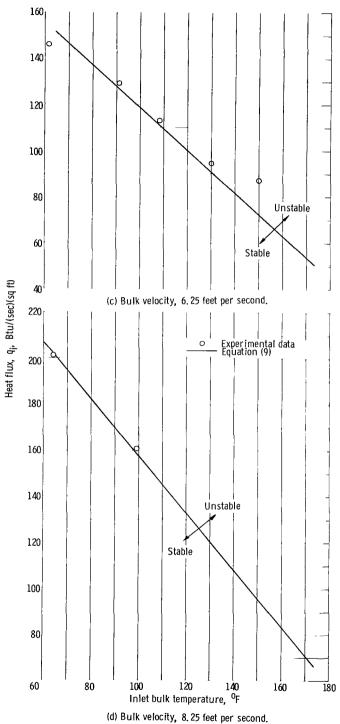


Figure 12. - Concluded.

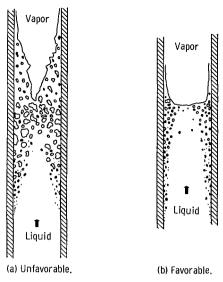


Figure 13. - Flow pattern development for inception of oscillations.

flow oscillations are to occur in subcooled boiling, this transition must be
abrupt, as illustrated in figure 13(b).
This means that if the flow oscillations
are to occur, the axial rate of change
of void fraction must be greater than
some minimum value, in addition to
satisfying the necessary criterion
given by equation (9). Further experimental evidence is necessary to
prove or disprove this argument,
which is based on limited experimental data and is presented here more
as a possibility than a fact.

## Effect of System Pressure

Similarly, the minimum heat flux necessary to cause flow oscillations was computed for constant inlet bulk temperature by varying the system pressure. The results are plotted in figure 14 together with experimental

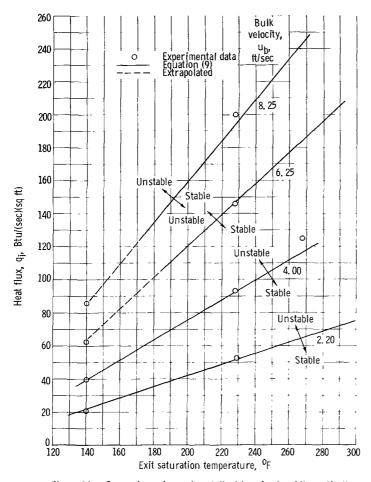


Figure 14. – Comparison of experimentally determined and theoretically predicted necessary heat flux for onset of oscillations as function of exit pressure (saturation temperature). Inlet plenum bulk temperature,  $\sim 70^{\circ}$  F.

results for four different velocities. Saturation temperature instead of pressure is used in the graph for convenience. The experimental and theoretical results show good correspondence. It should be noted (see table I, p. 8) that, at higher system pressure, the oscillations could not be obtained in the subcooled regime. It may be argued that the increase in heat flux necessary to obtain net quality at higher pressure is smaller than the corresponding decrease in the liquid-vapor density ratio. Thus, the net effect of pressure increase would be expected to be a lower axial rate of change of void fraction, which is an unfavorable condition for onset of flow oscillations in subcooled boiling.

The general trends of the effect of system pressure and bulk temperature found experimentally can also be predicted by examining the necessary criterion for inception of oscillations as given by equation (9).

With other conditions remaining constant, an increase in system pressure will decrease the value of the integral in equation (9) (since  $t_s$  increases with pressure) and thus result in a more stable system; a decrease in subcooling will result in a lower temperature profile  $t_s$ , which decrease the value of the integral, and thus render a more stable system.

## Effect of Boiler Length

A comparison between experimentally obtained and predicted heat flux at the onset of oscillations with a variable length test section is shown in figure 15. The results are in very good agreement and show an inverse proportionality between heat flux and length. It should be noted that as the length of the boiler tube decreases the total heat input necessary for inception of oscillations also decreases, which means that the exit tempera-

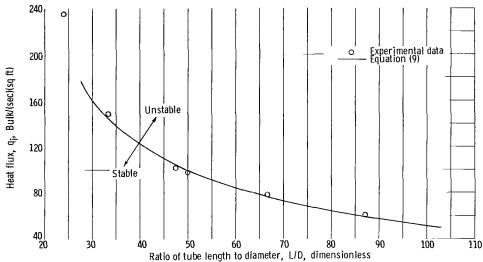


Figure 15. - Comparison of experimentally determined and theoretically predicted necessary heat flux for onset of oscillations as function of boiler length. Bulk velocity, 2.20 feet per second; inlet plenum bulk temperature, 70° F; exit plenum pressure, 20.0 psia.

ture decreases. In other words, a decrease in length will induce flow oscillations at somewhat lower exit bulk temperatures. The results shown in figure 15 were obtained at constant inlet bulk temperature and velocity and constant exit pressure.

## CONCLUDING REMARKS

Since the primary objective of this study was to determine the conditions at the onset of flow oscillations, little attention was paid to the amplitude and period of oscillations. In general, the amplitude was sufficiently high to allow the instability to be recognized. An examination of the traces revealed the frequency of oscillations to be directly proportional to the system pressure. The velocity, heat flux, and bulk temperature, however, seem to have very little, if any, effect on the frequency over the range of this investigation.

A few runs were conducted in which the heat flux was increased beyond the value corresponding to the onset of oscillations. It was found that by increasing the heat flux, a neutrally oscillating subcooled boiling system could become stable as the thermodynamic quality at the exit reached a positive value; increasing the heat flux further would cause the system to become unstable again in the net quality region. This phenomenon, obtained with an electrically heated tube, is illustrated in figure 16. It should be noted that the surface temperature oscillations in the two cases are different in nature. With net quality, the temperature is similar to an excursion, usually observed and associated with burnout and followed by a slow decrease. In the subcooled case, however, the temperature rises slowly and then suddenly drops at about the minimum flow, a behavior ob-

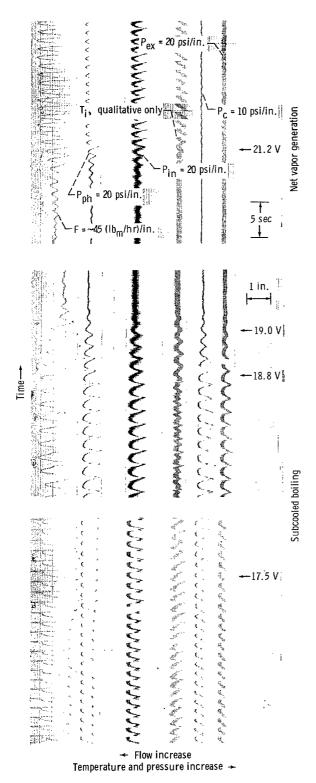


Figure 16. - Typical flow, pressure, and surface temperature behavior in transition region between subcooled boiling and net vapor generation.

served at the incipience of nucleate boiling. Some preliminary runs, in which an additional thermocouple further upstream was monitored on the oscillograph, yielded similar results: that is, conditions could be reached, such that the upstream surface temperature oscillated in a fashion characteristic of the incipience of boiling, while the exit thermocouple exhibited the burnout characteristics. Due to the oscillatory flow and therefore varying local quality, it is not surprising to attain this intermittent burnout phenomenon. It is believed that the periodic temperature excursions at the exit are the result of rather than the cause of flow oscillations. It is quite possible that with insufficient instrumentation the quickly developing oscillations can go undetected, resulting in what is usually referred to as "premature burnout" or "submaximum critical heat flux. "

Some additional runs were made with the annular test section where net quality was generated at the exit under stable conditions by sufficiently raising the inlet bulk temperature. It was observed that whereas the annular flow pattern did exit at the exit, the transition from bubbly to annular regimes was quite gradual. As the heat flux was increased, the transition moved upstream, became more abrupt, and finally resulted in oscillatory behavior. This can be explained by the fact that in the low quality region the pressure drop in the boiler increases with quality, with the downstream portion of the boiler contributing most to the increase. Consequently, the local pressure (and hence, the saturation temperature) in the upstream portion of the test section increased with heat flux. This in turn would suppress boiling in the upstream

section, which necessitates a higher axial rate of change of void fraction in the downstream part of the boiler.

It is believed that in net quality boiling, just as in the subcooled case, a critical value of thermal nonequilibrium between the two phases must be reached locally (not necessarily at maximum fluid enthalpy) if the oscillations are to occur. The fact that the system remained stable for all air injection tests also indicates this possibility. Further studies are necessary, however, to substantiate this theory and determine whether the thermal nonequilibrium is the driving force of flow oscillations.

### SUMMARY OF RESULTS

The criterion given by equation (9) predicted the necessary heat flux for incipience of oscillations occurring in low-pressure subcooled boiling systems. The necessary criterion was derived on the basis of local rates of bubble growth and collapse. The good agreement between the theoretical and experimental results indicates that the oscillatory flow behavior may be a strong function of thermal nonequilibrium between the liquid and vapor phases.

The experimental observations can be summarized as follows:

- 1. Basically, two types of oscillations were observed in subcooled boiling with electrically heated test sections: constant-amplitude oscillations and oscillations with increasing amplitude that resulted in surface temperature excursion. It is believed that the excursion was the direct result of the relatively low heat capacitance of the tube.
- 2. With the aid of high-speed movies, flow patterns at the inception of oscillations were studied. It was observed that when flow oscillations were present a quick transition from a bubbly type to an annular type of flow pattern existed in the test section. The interface oscillated up and down the tube with the same frequency as flow and pressure oscillations.
- 3. Comparable oscillatory behavior was obtained when boiling was simulated by steam injection through a porous wall. When two-phase flow was attained by injecting air through the porous wall, however, the system remained stable for all cases investigated.
- 4. An oscillatory, subcooled boiling system could be stabilized by increasing heat flux and thus generating net quality. The system remained nonoscillatory until some new heat flux value was reached. The surface temperature oscillations in subcooled and net quality vapor generation were entirely different in shape.

5. With other variables held constant, a decrease in system pressure and an increase in inlet subcooling tend to destabilize the subcooled boiling system.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, March 1, 1965.

## APPENDIX A

## SYMBOLS

c <sub>p</sub>	specific heat, Btu/(lb mass)(OF)	<b>y*</b>	wall distance parameter,
D	tube diameter, ft		$\frac{\sqrt{ au_{ ext{i}} ho_{ ext{i}}}}{ ext{y}}$ y, dimensionless
$^{ m h}_{ m fg}$	latent heat of evaporation,		$\mu_{\mathbf{i}}$
-6	Btu/lb mass	α	thermal diffusivity, sq ft/hr
K	constant	β	heat-transfer parameter,
k	thermal conductivity, Btu/(sec)(ft)(OF)		$\frac{q_i^{}}{c_p^{}t_i^{}\sqrt{ au_i^{} ho_i^{}}}$ , dimensionless
L	tube length, ft		-
m	constant	θ	time, hr
$N_{\mathbf{Pr}}$	Prandtl number, dimensionless	μ	viscosity, 1b mass/(sec)(ft)
${ m ^{N}_{Re}}$	Reynolds number, dimensionless	ζ	coefficient of bubble growth,
n	constant		dimensionless
P	pressure, psia	ρ	density, lb mass/cu ft
α .	heat flux, Btu/(sec)(sq ft)	au	shear stress, lb mass/(sec <sup>2</sup> )(ft)
$\mathbf{q_i}$	near max, Druy (sec)(sq m)		
ч <sub>і</sub> R	bubble radius, ft	Subscr	ipts:
-	bubble radius, ft	Subscr	ipts: average
R r	bubble radius, ft		
R	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r,	av	average
R r	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r, dimensionless	av b	average bulk
R r	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r,	av b c	average bulk condenser
R r r*	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r, dimensionless temperature, $^{\rm O}{\rm F}$ temperature parameter,	av b c ex	average bulk condenser exit plenum
R r r*	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r, dimensionless temperature, $^{\rm O}{\rm F}$ temperature parameter,	av b c ex i	average bulk condenser exit plenum inside wall
R r r*	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r, dimensionless temperature, $^{\rm O}{\rm F}$	av b c ex i in	average bulk condenser exit plenum inside wall inlet plenum
R r r*	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r, dimensionless temperature, ${}^0F$ temperature parameter, $\frac{(t_i - t)c_p \sqrt{\tau_i \rho_i}}{q_i}$ , dimensionless velocity, ft/sec	av b c ex i in l	average bulk condenser exit plenum inside wall inlet plenum liquid
R r r* t	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r, dimensionless temperature, ${}^0F$ temperature parameter, $\frac{(t_i - t)c_p \sqrt{\tau_i \rho_i}}{q_i}$ , dimensionless velocity, ft/sec	av b c ex i in l	average bulk condenser exit plenum inside wall inlet plenum liquid preheat
R r r* t t*	bubble radius, ft tube radius, ft tube radius parameter, $\frac{\sqrt{\tau_i \rho_i}}{\mu_i}$ r, dimensionless temperature, $^{\rm O}{\rm F}$ temperature parameter, $\frac{({\rm t_i-t}){\rm c_p}\sqrt{\tau_i \rho_i}}{{\rm q_i}}$ , dimensionless ${\rm q_i}$	av b c ex i in l ph s	average bulk condenser exit plenum inside wall inlet plenum liquid preheat liquid-vapor interface vapor

## 

#### APPENDIX B

#### UNIVERSAL VELOCITY AND TEMPERATURE DISTRIBUTION

In order to solve the differential equations for shear stress and heat transfer, Deissler assumed that eddy diffusivities for momentum and heat are equal. Furthermore, it was postulated that the shear-stress and heat-transfer distributions across the tube or boundary layers have a negligible effect on velocity and temperature distributions. The molecular shear-stress and heat-transfer terms in the equations were neglected in the region distant from the wall. When expressions for eddy diffusivities were assumed, Deissler arrived at expressions for nondimensional parameters for velocity and temperature distributions for a constant fluid properties case. The analysis is extended (refs. 8 and 10) to the case with variable viscosity, a property which for liquids varies more rapidly with temperature than other properties. For liquids, the variation of viscosity with temperature may usually be represented by

$$\frac{\mu}{\mu_i} = \left(\frac{t}{t_i}\right)^m \tag{B1}$$

where m is a constant. When  $t^*$  and  $\beta$  are defined as

$$t^* = \frac{(t_i - t)c_p\sqrt{\tau_i\rho_i}}{q_i}$$

and

$$\beta = \frac{q_i}{c_p t_i \sqrt{\tau_i \rho_i}}$$

equation (B1) becomes

$$\frac{\mu}{\mu_i} = (1 - \beta t^*)^{\mathrm{m}} \tag{B2}$$

When equation (B2) in the shear-stress and heat-transfer equations is used together with all the assumptions stated previously, Deissler's universal velocity and temperature profiles for  $y^* < 26$  can be written in integral form as

$$u^* = \int_0^{y^*} \frac{dy^*}{(1 - \beta t^*)^m + n^2 u^* y^*} \left\{ 1 - \exp\left[ -\frac{n^2 u^* y^*}{(1 - \beta t^*)^m} \right] \right\}$$
(B3)

and

|'-

$$t^* = \int_0^{y^*} \frac{dy^*}{\left(\frac{1}{(N_{Pr})} + n^2 u^* y^* \left\{1 - \exp\left[-\frac{n^2 u^* y^*}{(1 - \beta t^*)^m}\right]\right\}}$$
(B4)

where t\*, u\*, y\*, and  $\beta$  are dimensionless temperature, velocity, distance, and heat-transfer parameters, respectively, and n is a constant.

For the region at a distance from the heated wall, that is, for  $26 < y^* \le r_1^*$ , equations containing variable viscosity are neglected and one obtains for universal velocity and temperature distributions in this region the following expressions:

$$u^* = u_{y^*=26}^* + \frac{1}{K} \ln \frac{y^*}{26}$$
 (B5)

and

$$t^* = t^*_{y^*=26} + u^* - u^*_{y^*=26}$$
 (B6)

where K is a constant.

Equations (B3) and (B4) can be solved simultaneously by iteration. Thus, the solution of equations (B3) to (B6) will yield the universal velocity and temperature distributions in the tube for any set of conditions.

The temperature parameter resulting from equations (B4) and (B6) is defined as

$$t^* = \frac{(t_i - t)c_p \tau_i}{q_i \sqrt{\tau_i/\rho_i}}$$
 (B7)

which implies that the shear stress at the wall  $\tau_i$  must be known before the temperature t can be determined. This can be accomplished by using the 1/7 power law to determine the initial value of  $\tau_i$ , which was used in the initial value of the heat-transfer parameter  $\beta$  in equations (B3) and (B4). Simultaneous iteration of these equations yielded the universal distributions for the assumed values of  $\tau_i$ . From equation (B7) and the definition of  $t_b^*$  an expression for shear stress at the wall is desired:

$$\tau_{i} = \frac{1}{\rho_{i}} \left[ \frac{q_{i} t_{b}^{*}}{c_{p} (t_{i} - t_{b})} \right]^{2}$$
(B8)

where

$$t_{b}^{*} = \frac{\int_{0}^{r_{1}^{*}} t^{*}u^{*}(r_{1}^{*} - y^{*})dy^{*}}{\int_{0}^{r_{1}^{*}} u^{*}(r_{1}^{*} - y^{*})dy^{*}}$$
(B9)

From equations (B8) and (B9) a new value of  $\tau_i$  is calculated, and it is used for calculating the new value of  $\beta$ . The process is repeated until the calculated and assumed values of  $\tau_i$  correspond within 0.01 percent. The final values of  $\tau_i$  and t\* are then used to calculate the temperature profile t from equation (B7).

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